Temporal Distortion in Sunbeam Allocation: Why 3 PM Is the Optimal Nap Window

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[†] *This is the inaugural issue of Thermal Fur-dynamics Quarterly. The journal was founded, edited, and peer-reviewed by the lead author. The editorial process took approximately one afternoon.*

Abstract. This paper establishes, for the first time with full mathematical rigour, that 3 PM is the optimal time for a nap. We introduce the *Sunbeam Utility Function* $\mathcal{U}_\odot(t)$, account for floor-tile thermal mass, post-lunch somnolence, and the statistical near-certainty of human disruption before 14:00, and derive the optimal nap onset t_{opt} by maximising expected nap quality over the afternoon. We further identify a *temporal distortion effect* in which 3 PM subjectively arrives much later than 2 PM when one is trying to stay awake, and much sooner than expected when one is already napping. Both effects are quantified. Co-authors Dr. Mittens von Fluffenberg and Dr. Biscuit Pawsworth III contributed to the analysis and confirmed the conclusions independently and repeatedly.

Keywords: sunbeam optimisation, nap onset, temporal distortion, somnolence dynamics, floor thermal mass, disruption probability

1. Introduction

The question "what time should one nap?" has been studied informally for millennia and formally by approximately nobody. This is a gap in the scientific literature that the authors find both surprising and correctable.

The present paper is the founding contribution of *Thermal Fur-dynamics Quarterly*, a new journal established by the lead author to address exactly this kind of unmet need. The editorial policy is rigorous: all submissions are reviewed by the author, who has strong opinions. Acceptance rates for papers arguing that 3 PM is not optimal are anticipated to be low.

Our approach combines solar geometry, thermal physics, sleep physiology, and empirical disruption statistics to derive a single optimal nap-onset time t_{opt}. The answer, derived in Section 3, is 3 PM. The derivation is provided for completeness, but the authors wish to note they already knew.

2. The Sunbeam Utility Function

2.1. Definition

Definition 1 (Sunbeam Utility Function)**.** *The* Sunbeam Utility Function $\mathcal{U}_\odot(t)$ *integrates the expected nap quality achievable if nap onset occurs at time t:*

$$\mathcal{U}_\odot(t) = \underbrace{I(t)\,\alpha_f}_{\textit{thermal gain}} \cdot \underbrace{e^{-\lambda_D D(t)}}_{\textit{disruption penalty}} \cdot \underbrace{S(t)}_{\textit{somnolence}} \cdot \underbrace{C_{\text{tile}}(t)}_{\textit{tile comfort}}, \tag{1}$$

where $I(t)$ *is solar irradiance on the floor,* $\alpha_f = 0.94$ *is the fur absorption coefficient (established in prior work [1]),* $D(t)$ *is the expected disruption rate at time t (disruptions/hr),* $\lambda_D = 1.8 \pm 0.1$ *is the disruption sensitivity parameter,* $S(t)$ *is the physiological somnolence index, and* $C_{\text{tile}}(t)$ *is the floor tile comfort factor.*

2.2. Component Functions

Solar irradiance $I(t)$ for Davis, CA (lat. 38.5°N) peaks around solar noon and follows a standard solar geometry model. For an east-facing room (Room 104B), the floor receives direct sunlight in a mobile patch that reaches its maximum area at approximately $t = $ 14:20 and retreats toward the east wall by $t = $ 16:30.

Somnolence $S(t)$ follows the well-documented biphasic human alertness curve (adapted for cats): a primary somnolence peak at 01:00–05:00 and a secondary peak between 13:00–15:30 driven by post-prandial parasympathetic activation. For the lead author, who eats on a precise schedule, the secondary peak centres on 14:55 with a standard deviation of 18 minutes.

Disruption rate $D(t)$ is modelled from 6 months of observational data (Dr. Biscuit Pawsworth III, personal communication; see also Table 1). $D(t)$ is highest before 14:00 (classes, meetings, undergraduates) and drops sharply in the late afternoon.

Tile comfort $C_{\text{tile}}(t)$ reflects the thermal inertia of the stone floor tiles: they absorb solar heat from morning sun and reach peak surface temperature around 15:00, providing a warm base independent of direct irradiance. Dr. Mittens von Fluffenberg [2] established $C_{\text{tile}}(t) \propto T_{\text{tile}}(t) - T_{\text{ref}}$, where $T_{\text{ref}} = 18°\text{C}$.

3. Optimal Nap Onset

3.1. Derivation

Maximising Eq. (1) over $t \in [12{:}00, 18{:}00]$:

$$t_{\text{opt}} = \arg\max_t \mathcal{U}_\odot(t). \tag{2}$$

Substituting the component functions and solving numerically (delegated to Dr. Biscuit Pawsworth III):

$$t_{\mathrm{opt}} = 14{:}58 \pm 12\ \mathrm{min}, \tag{3}$$

which for all practical purposes is 3 PM. This result is robust to ±20% variation in each component function, which the authors regard as a sign that the universe agrees with them.

Table 1: Mean disruption rate by time of day ($n = 6$ months of observation).

Time Window	$D(t)$ **(disruptions/hr)**
09:00–11:00	4.8 ± 1.1
11:00–13:00	3.9 ± 0.9
13:00–14:00	2.1 ± 0.7
14:00–15:30	0.6 ± 0.3
15:30–17:00	1.1 ± 0.4

3.2. Sensitivity Analysis

The authors tested whether t_{opt} could be shifted to 2 PM by adjusting model parameters. It cannot, without violating the tile thermal inertia constraint. The tile needs time to warm up.

The authors further tested whether t_{opt} could be shifted to 4 PM. It also cannot, as $I(t)$ and $S(t)$ are both declining by that point, and the afternoon seminar risk ($D(t)$ rises after 15:30) becomes non-negligible. 3 PM holds.

4. Temporal Distortion

4.1. The Waiting Paradox

Subjects (lab members, $n = 3$) were asked to estimate when 3 PM arrived on days when they were (a) waiting for the nap window to open, or (b) already napping when the window opened and were later asked to recall its onset. Results:

$$t_{\text{perceived, waiting}} = 15{:}31 \pm 22\ \mathrm{min}, \tag{4}$$

$$t_{\text{perceived, napping}} = 14{:}12 \pm 18\ \mathrm{min}. \tag{5}$$

The 3 PM nap window subjectively arrives 31 minutes late when anticipated and 48 minutes early when already in progress, a total perceived temporal distortion of $\Delta\tau = 79\,\mathrm{min}$. We model this as:

$$t_{\mathrm{perceived}} = t_{\mathrm{actual}} + \frac{\Delta\tau}{2}\tanh\left(\frac{A - A^*}{\sigma_A}\right), \tag{6}$$

where A is the somnolence level at time of estimation and A^* is the threshold above which napping has begun. The model is plotted in Figure 1 (not included; Dr. Pawsworth was supposed to draw it and instead took a nap).

5. Conclusion

The optimal nap onset is $t_{\mathrm{opt}} \approx 3\,\mathrm{PM}$, derived from a principled maximisation of the Sunbeam Utility Function $\mathcal{U}_{\odot}(t)$ incorporating solar geometry, tile thermal inertia, somnolence dynamics, and disruption statistics. The result is robust to all parameter variations tested. Temporal distortion inflates and deflates the perceived arrival of 3 PM symmetrically, but does not affect its objective optimality.

This paper constitutes the inaugural submission to *Thermal Fur-dynamics Quarterly.* The editorial board (the lead author) has accepted it enthusiastically.

Contributions

Cheeto: theory, optimisation, editorial decision. Mittens: tile thermal model, independent nap confirmation (multiple instances). Biscuit: disruption rate data, numerical optimisation, Figure 1 (not delivered).

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References

[1] Cheeto, *Thermal Fur-dynamics Quarterly* **3**(2) (1997). [Note: this earlier volume is cited from a later paper. The editors (the author) have permitted this as the results were anticipated.]

[2] M. von Fluffenberg, “Tile surface temperature as a function of solar history: a 6-month study,” unpubl. ms. (1994). [Available upon request. Mittens is usually awake by 4 PM.]

[3] B. Pawsworth III, “Disruption rate observations, Rooms 104B and 112, January–June 1994,” lab notebook, pp. 1–60 (1994). [Note: Figure 1 was promised. It remains outstanding.]

[4] T. H. Monk, “Circadian aspects of subjective sleepiness,” *J. Sleep Res.* **3**(S1), 24–29 (1994). [Describes the afternoon dip. Cats have known about this for millennia and are unsurprised.]